**ASSIGNMENT 2 Due: Tuesday 4th October 2021**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ID#: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Write your answers in the space provided.**

**Full marks will not be awarded if full working/explanation is not shown.**

**Attach the R code and Assignment as SEPARATE files. You should NOT use any R code that may be freely available on the internet.**

**You should NOT write R code or any output …paste it from R. You can write any explanations in the spaces provided.**

**Upload (1) the R code script file and (2) the assignment (as a PDF) in myelearning.**

**Question 1:**

Suppose that are i.i.d. Poisson random variables whose distribution is a mixture of Poisson random variables with parameters . We observe a Poisson random variable with mean with probability and a Poisson variable with mean with probability .

1. Derive the EM Algorithm to estimate the parameters. **[10]**
2. Suppose the following data are observed 1, 2, 3, 8 and 12 perform one iteration (BY HAND!!) of the EM algorithm to estimate the parameters You can use an ordinary calculator OR the calculator facilities in R. **[10]**
3. Write R code to perform the EM Algorithm and include in the R code an appropriate stopping tolerance level. **[10]**
4. Generate 100 values from a Poisson distribution with mean 5 and 900 values from a Poisson distribution with mean 10. Amalgamate those 1000 values into a single column. Then, run the R code you constructed to see how well your algorithm is able to estimate the parameters.  **[10]**

**Question 2:**

(a)Show that the function illustrated in Table 1 represents the joint probability mass function of the two discrete random variables X and Y? Justify your answer. ( 2 marks)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| f(x,y) |  | **x** | | |
|  |  | **1** | **2** | **3** |
| y | **1** | 1/20 | 2/20 | 1/20 |
| **2** | 4/20 | 7/20 | 5/20 |

**Table 1**

1. Find E(XY) by hand. Leave your answer in exact form. (3 marks)

(c) Write an appropriate algorithm to generate pairs of random variables X and Y from the joint probability mass function in Table 1. (5 marks)

State the Algorithm

1. (i) Implement the algorithm in R and generate 100,000 pairs of random variables (X,Y) from the distribution.

(PASTE R CODE) (5 marks)

(ii) Estimate E(XY) from the 100,000 pairs of values you simulated (USE R!). (3 marks)

(iii) Compare the estimated mean of E(XY) with the true mean you found in (b). (2 marks)